Dear Learner:

These are the full solutions to most of the questions in your classroom booklet. If the solution to a question you are looking for is NOT here, that could be because it is one of the designated "extra questions" – meaning you were given the solutions in-class. If you see any mistakes here – please email feedback@rtdlearning.com! Thanks..

Unit 1 – Function Operations and Transformations

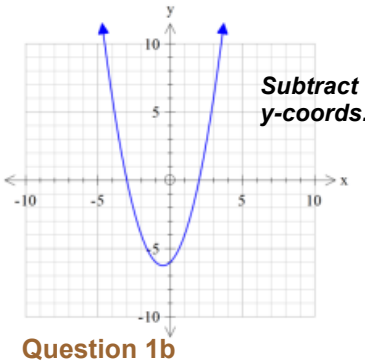
RTD EXAM PREP

SOLUTIONS

Question 1a

$h(x) = (x^2 - 4) - (2 - x)$

$= \boldsymbol{x^2 + x - 6}$



Subtract the y-coords.

Question 1b

Divide the y-coords

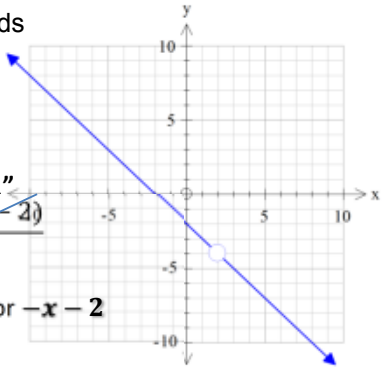
Domain: $x \neq 2$

$$h(x) = \frac{x^2 - 4}{2 - x}$$

"-1"

$$h(x) = \frac{(x+2)(x-2)}{2-x}$$

$h(x) = -(x+2)$ or $\boldsymbol{-x-2}$

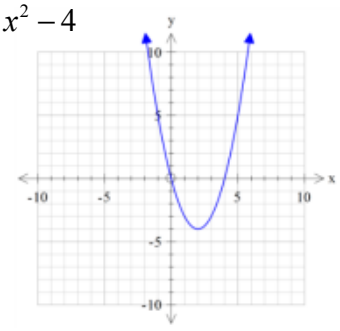


Question 1c

$f(g(x)) = (2-x)^2 - 4$

$= 4 - 4x + x^2 - 4$

$= x^2 - 4x$



Question 1d

$f(f(x)) = (x^2 - 4)^2 - 4$

$f(f(0)) = ((0)^2 - 4)^2 - 4$

$= 16 - 4$

$= \boldsymbol{12}$

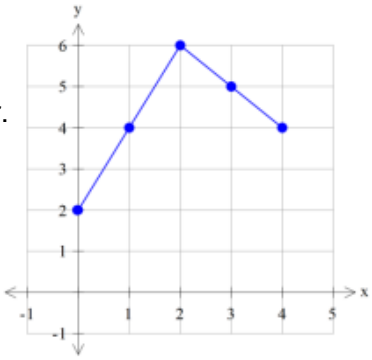
Question 2

"Extra Question" – solutions given in class

Question 3

$h(x) = f(x) + g(x)$

Add y-coords together.



Question 4

Test each point by subtracting the y-coords at each given x-coord.

Point 1 (2,0)→ Yes. At x=2 on f(x) the point is (2,0). On g(x) the point is (2,0). 0 – 0 = 0 so (2,0) is on h(x).

Point 2 (0, -5)→ Yes. On f(x) the point is (0,-2). On g(x) the point is (0,3). -2 -3 = -5 so (0, -5) is on h(x).

Point 3 (-2, 8) → No.

Point 4 (-4, -5) → Yes.

Answer: ***1,2, and 4.***

Question 5

$(x, y) \rightarrow (x-3, -y+1)$

$(3,9) \rightarrow (3-3, -(9)+1) \rightarrow (0, -8)$

Question 6

$g(x) = f(x+4) - 3$

left 4 and down 3

$\therefore$ C

Question 7

Reflection about the line y = 0. (vert.)

Make entire expression negative!

$y = -(\sqrt{x-3}+1)$

$y = -\sqrt{x-3}-1$

D: $\{x \geq 3\}$ **R:** $\{y \leq -1\}$

Reflection about the line x = 0 (horiz)

Replace x with "$-x$"

$y = \sqrt{(-x)-3}+1$

$y = \sqrt{-(x+3)}+1$

D: $\{x \leq -3\}$ **R:** $\{y \geq 1\}$

Question 8

$y = f(-x)$ is a horizontal reflection (about y-axis) – which is **graph 4**.

$y = f^{-1}(x)$ is the inverse, all points (x, y) become (y, x)…so "start point" on $f(x)$, $(2, 3)$ become $(3, 2)$…entire graph is a reflection about line $y = x$, which is **graph 3**.

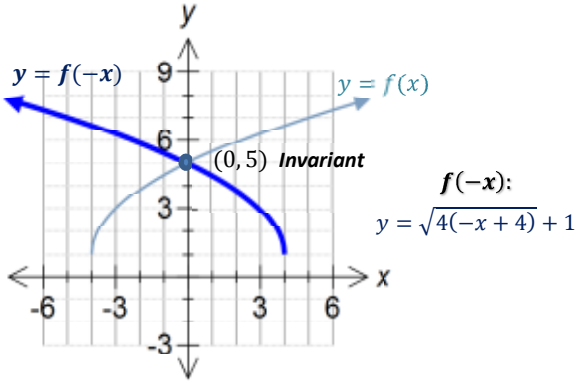
ANSWER: **C**

Questions 9-11

"Extra Questions" – see book of answers

Question 12

All points $(x, y) \rightarrow (-x, y)$, so start point $(-4, 1)$ becomes $(4, 1)$ and graph goes LEFT. (not right)



Question 13

Stretched about the line y = 0 by a factor of 3.

$y = 3f(x)$

$f(x) = 3(x^2 - x - 6)$

$f(x) = 3x^2 - 3x - 18$

#13 (continued) Reflected about the y-axis.

$(x, y) \rightarrow (-x, y)$

$f(x) = (-x)^2 - (-x) - 6$

$f(x) = x^2 + x - 6$

Question 14

Vertical Stretch Factor of 5, Horizontal Reflection and Translated up 1.

$(x, y) \rightarrow (-x, 5y+1)$

$(-2, 8) \rightarrow (-(-2), 5(8)+1) \rightarrow (2, 41)$

Question 15

"Extra Question" – solutions given in class

Question 16a

The "min" of the parabola $f(x)$ is -4, after a vertical stretch (invariant points are on the x-axis) the min of $g(x)$ is 2. So….vertical stretch, factor of 2.

$g(x) = \frac{1}{2} f(x)$

$g(x) = \frac{1}{2}\left[(x-1)^2 - 4\right]$

$g(x) = \frac{1}{2}(x-1)^2 - 2$

Question 16b

The "start point" of the graph $f(x)$ is at $(-1, -3)$, after a horiz.stretch (invariant point is on the y-axis) the start point of $g(x)$ is at $(-4, -3)$ or 4 times as far horizontally from the y-axis. So….horiz.stretch, factor of 4.

$g(x) = f(\frac{1}{4}x)$

$g(x) = \sqrt{\frac{1}{4}x+1} - 3$ or $g(x) = \sqrt{\frac{1}{4}(x+4)} - 3$

Question 16c

The vertex of the graph $f(x)$ is at $(3,0)$, after a horiz.stretch (invariant point is on the y-axis) the start point of $g(x)$ is at $(1,0)$ or 1/3rd as far horizontally from the y-axis. So….horiz.stretch, factor of 1/3.

$g(x) = f(3x)$

$g(x) = |3x - 3|$

Question 17

"Extra Question" – sol. handout given in class

Question 18

The lowest y-value on the graph, no matter the x-value will be at 4. So if that point is vertically stretched by a factor of 2, the y-value becomes 8. The x-value will remain the same so it doesn't matter what it is. Then that y-value at 8 will be translated down 9 units, which will put it at -1.
ANSWER: **A**

Question 19 ANSWER: **D**

$$(x,y) \to (4x, -\frac{1}{3}y)$$

$$(-3,6) \to (4(-3), -\frac{1}{3}(6)) \to (-12,-2)$$

Question 20

"Extra Question" – sol. handout given in class

Question 21

First – re-write…

$g(x) = 2f(x-3) - 4$

VSF of 2: affects range

Translated right 3: affects domain

Translated down 4: affects range

So domain moves right 3. So new domain is [2,6]

Range gets multiplied by 2 then moved down 4. So new range is [0,8]

$\boldsymbol{a = 2, b = 6, c = 0}$, and $\boldsymbol{d = 8}$.

Question 22

"Extra Question" – sol. handout given in class

Question 23

$f(x) = \sqrt{x+3} - 1$

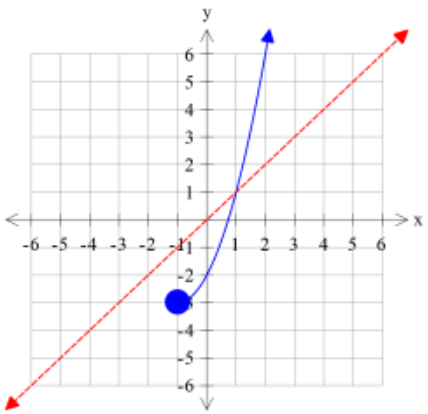
$y = \sqrt{x+3} - 1$

$x = \sqrt{y+3} - 1$

$x + 1 = \sqrt{y+3}$

$(x+1)^2 = y+3$

$y = (x+1)^2 - 3, x \geq 1$



Question 24

First graph is a function and passes horizontal line test so inverse will also be a function.

Second graph is a function but does not pass horizontal line test so the inverse will not be a function.

Third graph does not pass vertical line test so it is not a function. It does pass the horizontal line test so the inverse will be a function.

Question 25

"Extra Question" – sol. handout given in class

Question 26

$y = -f(x) \to$ Vertical Reflection

Invariant points are x-ints.

$\therefore$ point 5

$y = f(-x) \to$ Horizontal Reflection

Invariant points are y-ints.

$\therefore$ point 3

$x = f(y) \to$ Inverse

$\therefore$ point 1

(For inverse invariant pts are on line $y = x$)

Question 27

B

Vertex of $f(x)$ is at (0, 4) Restrict domain to form a "half-parabola", so that the inverse passes the vertical line test.

SOLUTIONS

RTD EXAM PREP

Unit 2 – Polynomial Functions

Question 1

$\{\pm1, \pm2, \pm3, \pm6\}$

$P(-1) = (-1)^3 + 2(-1)^2 - 5(-1) - 6$

$p(-1) = 0$

$\therefore x+1$ is a factor.

$$-1 \left| \begin{array}{cccc} 1 & 2 & -5 & -6 \\ & -1 & -1 & 6 \\ \hline 1 & 1 & -6 & 0 \end{array} \right.$$

$(x+1)(x^2 + x - 6) = 0$

$(x+1)(x+3)(x-2) = 0$

$x = -1, -3, 2$

Question 2 *See other solution sheet.*

Question 3

Can graph and look at roots or test each factor.

$P(-2) = 0$

$P(4) \neq 0$

$P(6) \neq 0$

$P(-8) \neq 0$

$\therefore A$

Question 4

$$-3 \left| \begin{array}{cccc} 2 & 7 & -2 & -15 \\ & -6 & -3 & 15 \\ \hline 2 & 1 & -5 & 0 \end{array} \right.$$

$2x^2 + x - 5$

$\therefore a = 2, b = 1, c = 5$

Question 5 *See other solution guide*

Question 6

1: **Yes** because all exponents are positive integers

2: No because there is a negative exponent

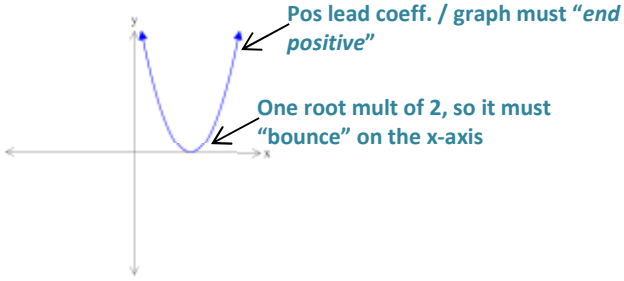
3: **Yes** because the exponent on the variable is a positive integer

4: No because one of the terms is being divided by a variable

5: No because one of the terms has a variable as an exponent.

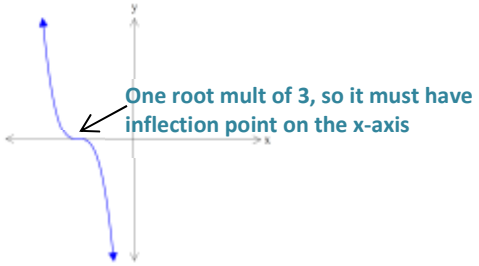
Question 7a

Answers will vary



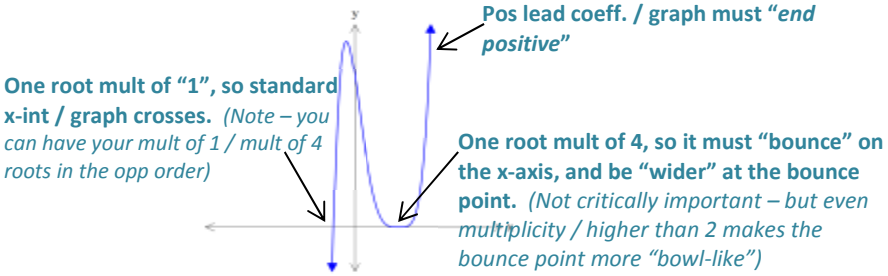
Question 7b

Answers will vary



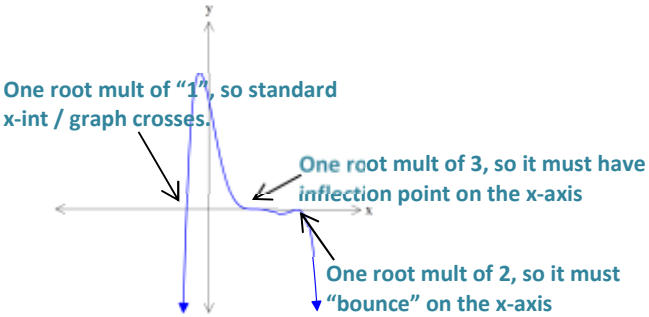
Question 7c

Answers will vary. *(order can be different!)*

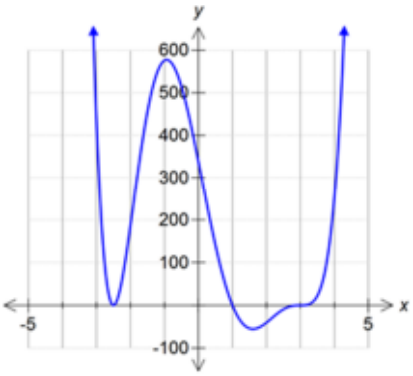


Question 7d

Answers will vary. *(order can be different!)*



Question 8



X-Intercepts: (-2.5,0), (1,0), (3,0)

Y-Intercept: (0,337.5)

Question 8b

$D: x \in R$

$R: y \geq -55.4$

Question 8c

x = 1, -5/2, 3

Question 9

Possible multiplicity of 2 (right x-int) and a multiplicity of 1. Minimum degree is 3.

$y = a(x-2)(x+1)^2$

Sub in y-int (0, -8) off graph for x & y.

$-8 = a(0-2)(0+1)^2$

$-8 = a(-2)(1)$

$-8 = -2a$

$4 = a$

$y = 4(x+1)^2(x-2)$

Question 10 *See other solution sheet*

Question 11

*Positive lead coefficient: Graph 4

*Two different zeroes, ea with multiplicity 2: Graph 2

*Degree 4 function: Graph 3

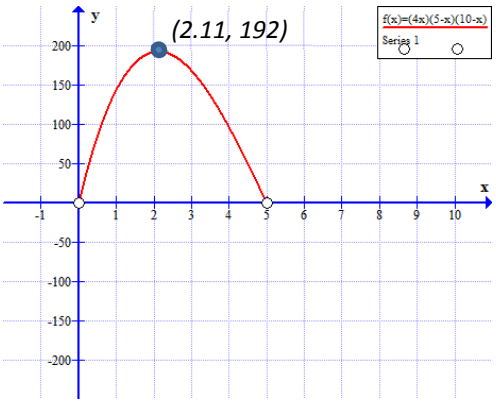
Question 12a

$V = x(10x-2x)(20-2x)$

$V = 4x(5-x)(10-x)$

Question 12b

$0 < x < 5;$



Question 12c

2nd→calc→max

X=2.11 cm (Gives max volume of 192 cm^3)

Question 13,14,15

See other solution sheet

Unit 3 – Exponential/Log. Functions

SOLUTIONS

Question 1a

$5^{(3x-8)} = (5^2)^{2x}$

$5^{(3x-8)} = 5^{4x}$

$3x-8 = 4x$

$x = -8$

Question 1b

$\left[(x-2)^{\left(-\frac{2}{3}\right)}\right]^{-\frac{3}{2}} = (25)^{-\frac{3}{2}}$

$x-2 = \left(\frac{1}{25}\right)^{\frac{3}{2}}$

$x-2 = \left(\pm\sqrt{\frac{1}{25}}\right)^3$

$x-2 = \left(\frac{1}{5}\right)^3 \quad or \quad x-2 = \left(-\frac{1}{5}\right)^3$

$x-2 = \frac{1}{125}$ | $x-2 = -\frac{1}{125}$

$125x-250 = 1$ | $125x-250 = -1$

$125x = 251$ | $125x = 249$

$x = \frac{251}{125}$ | $x = \frac{249}{125}$

Question 1c

$2^{(x-1)} = (2^6)^{(x+1)}(2^x)$

$2^{(x-1)} = (2)^{(6x+6)}(2^x)$

$2^{(x-1)} = (2)^{(6x+6+x)}$

$2^{(x-1)} = (2)^{(7x+6)}$

$x-1 = 7x+6$

$-7 = 6x$

$x = -\frac{7}{6}$

Question 2a

$y = b^x$

Substitute in point from graph for x and y

$(1,4)$

$4 = b^1$

$\therefore b = 4$

$\therefore y = 4^x$

Question 2b

$y = b^x$

Substitute a point from graph

for x and y; $(-1,3)$

$3 = b^{-1}$

$3 = \frac{1}{b}$

$3b = 1$

$b = \frac{1}{3}$

$\therefore y = \left(\frac{1}{3}\right)^x$

Question 3

$k = -2$

$y = a(2^x) - 2$

Substitute a point from graph

for x and y; (1,4)

$4=a(2^1) - 2$

$6 = 2a$

$3 = a$

$\therefore y = 3(2)^x - 2$

Question 4a

$a = 80$

The material decays from

80 to 40 counts in 2 days.

b=$\frac{1}{2}$ $\qquad p = 2\,days$

$\therefore C = 80\left(\frac{1}{2}\right)^{\frac{t}{2}}$

Domain: $\{t \geq 0\}$ Range: $\{0 < C \leq 80\}$

Question 4b

$C = 80\left(\frac{1}{2}\right)^{\frac{1.5}{2}}$

$C \approx 48$

Question 4c

$C = 0.85^t$

$b = 1 - rate\ of\ decay$

Questions 5, 6 – *See other handout*

Question 7

$y-\text{int}:(0,y)$

$y=a^{0+1}+b$

$y=a+b$

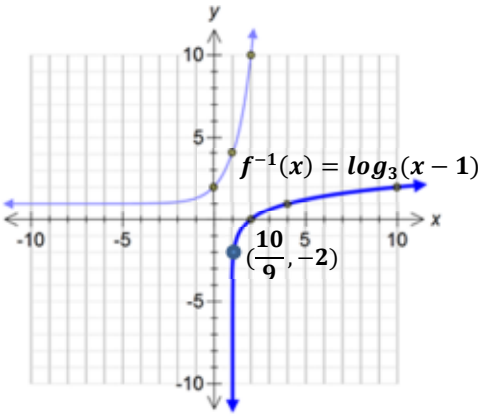
$\therefore D$

Question 8

Statement 1: **False**
Statement 2: **True**
Statement 3: **True**
Statement 4: **False**
Statement 5: **True**
Statement 6: **False**

$\therefore 2,3,and,5$

Question 9a



Question 9b

$y=3^x+1$

$x=3^y+1$

$x-1=3^y$

$y=\log_3(x-1)$

Question 9c

$f(x)$	$f^{-1}(x)$
$D:\{x\in R\}$	$D:\{x>1\}$
$R:\{y>1\}$	$R:\{y\in R\}$

Question 10a

$x=\left(\frac{1}{3}\right)^{-3}$

$x=\left(\frac{3}{1}\right)^{3}$

$x=27$

Question 10b

$\left(x^{\frac{4}{3}}\right)^{\frac{3}{4}}=(16)^{\frac{3}{4}}$

$x=\left(\sqrt[4]{16}\right)^{3}$

$x=(2)^{3}$

$x=8$

Question 10c

$16^x=2\sqrt{8}$

$(2^4)^x=2^1\cdot(2^3)^{\frac{1}{2}}$

$2^{4x}=2^1\cdot 2^{\frac{3}{2}}$

$2^{4x}=2^{\frac{3}{2}+1}$

$2^{4x}=2^{\frac{5}{2}}$

$4x=\frac{5}{2} \quad >> \quad 8x=5 \quad >> \quad x=\frac{5}{8}$

Question 11a

$\log_5 125=3$

Question 11b

$\log 0.01=-2$

Question 11c

$\log_m n=x$

Question 12, 13, 14

See other sol'n sheet

Question 15

$x^{-\frac{3}{2}}=\frac{1}{64}$

$\left(x^{-\frac{3}{2}}\right)^{-\frac{2}{3}}=\left(\frac{1}{64}\right)^{-\frac{2}{3}}$

$x=64^{\frac{2}{3}} \quad ... \quad x=\left(\sqrt[3]{64}\right)^{2}$

$x=4^2 \quad ... \quad x=16 \quad ... \quad \therefore A$

Question 16

$y = \log_b(3(x+4))$

$left 4$

$\therefore x = 0 \, moves \, to \, x = -4$

$\therefore A$

Question 17

$x = 3^y$

$y = \log_3 x$

$\therefore A$

Question 18

$3^6 = a,\ 3^5 = b$ ... so $a = 729, b = 243$

$log_3(9 * 729 * 243^2)$

$= log_3 9 + log_3 729 + log_3 243^2$

$= log_3 9 + log_3 729 + 2log_3 243$

$= 2 + 6 + 2 * 5$ ➔ $= \mathbf{18}$

Question 19a

$$\log\left(\frac{40 \square 500}{2}\right)$$

$$\log 10000$$

$$= 4$$

Question 19b

$$\log_3 144^{\frac{1}{2}} - \log_3 4 + 2(1)$$

$$\log_3 \sqrt{144} - \log_3 4 + 2$$

$$\log_3 12 - \log_3 4 + 2$$

$$\log_3\left(\frac{12}{4}\right) + 2$$

$$\log_3 3 + 2$$

$$1 + 2$$

$$= 3$$

Question 19c

$$\log A^2 + \log C^{\frac{1}{3}} - \log C^2 + \log B^3$$

$$\log\left(\frac{A^2 C^{\frac{1}{3}} B^3}{C^2}\right)$$

$$\log\left(\frac{A^2 B^3}{C^{\frac{5}{3}}}\right)$$

Question 19d(i)

$\log(8 * 100)$

$= log_{10} 8 + log_{10} 100$ ➔ $m + 2$

Question 19d(ii)

$$\log 512^{\frac{1}{2}}$$

$$\log(8^3)^{\frac{1}{2}}$$

$$\log(8)^{\frac{3}{2}}$$

$$\frac{3}{2}\log 8 \quad ... \ = \frac{3}{2}m$$

Question 19e(i)

$log_2(9 * 18)$ *(Since* $9 * 18 = 162$*)*

$= log_2 9 + log_2 18$

$= log_2 9 + log_2(9 * 2)$

$= log_2 9 + log_2 9 + log_2 2$

$= x + x + 1$ ➔ $= \mathbf{2x + 1}$

Question 19e(ii)

$$\log_2(9)^3 - \log_2(2)^{\frac{1}{2}}$$

$$3\log_2 9 - \frac{1}{2}\log_2 2$$

$$= 3x - \frac{1}{2}(1)$$

$$= 3x - \frac{1}{2}$$

Question 19f(i)

$$\log_5 5 + \log_5(x)^3$$

$$1 + 3\log_5(x)$$

$$= 1 + 3(2)$$

$$= 7$$

Question 19f(ii)

$$\log_5(x)^2 - \log_5 25$$

$$2\log_5(x) - \log_5(5)^2$$

$$2(2) - 2$$

$$= 2$$

Question 20

See additional soln's sheet

Question 21

$$\frac{\log\sqrt{200}}{\log 5}$$
$$=1.65$$

Question 22

$\frac{logx}{log7}=\frac{log60}{log4}$ ← *use change of base, then cross-mult*

$(logx)(log4)=(log7)(log60)$

$log_{10}x=\frac{log7*log60}{log4}$

$10^{(log7*log60)/log4}=x$

$x\approx \mathbf{313.29}$

Question 23

$$\log_3 y+\log_3 x=c$$
$$\log_3(yx)=c$$
$$3^c=yx$$
$$\frac{3^c}{x}=y$$
$$\therefore D$$

Question 24

$$\log x^2-\log z^{\frac{1}{2}}+\log y^3$$
$$\log\left(\frac{x^2y^3}{z^{\frac{1}{2}}}\right)$$
$$\log\left(\frac{x^2y^3}{\sqrt{z}}\right)$$
$$\therefore A$$

Question 25 *See other sol'n sheet*

Question 26

$$\log_3^{\frac{1}{3}}+\log_3 x$$
$$\log_3 3^{-1}+\log_3 x$$
$$=-1+15$$
$$=14$$
$$\therefore A$$

Question 27 *See other sol'n sheet*

Question 28a

$$1.2^{(2x+1)}=15$$
$$\log 1.2^{(2x+1)}=\log 15$$
$$(2x+1)\log 1.2=\log 15$$
$$2x\log 1.2+1\log 1.2=\log 15$$
$$2x\log 1.2=\log 15-\log 1.2$$
$$x=\frac{\log 15-\log 1.2}{2\log 1.2}\rightarrow Exact$$
$$x\approx 6.93$$

Question 28b

$$\log 2^{(x+3)}=\log 17^x$$
$$(x+3)\log 2=x\log 17$$
$$x\log 2+3\log 2=x\log 17$$
$$x\log 2-x\log 17=-3\log 2$$
$$x(\log 2-\log 17)=-3\log 2$$
$$x=\frac{-3\log 2}{\log 2-\log 17}\rightarrow Exact$$
$$x\approx 0.97$$

Question 28c

$$10=100\left(\frac{1}{2}\right)^{\frac{22.4}{p}}$$
$$0.10=\left(\frac{1}{2}\right)^{\frac{22.4}{p}}$$
$$\log 0.10=\log\left(\frac{1}{2}\right)^{\frac{22.4}{p}}$$
$$\log 0.10=\frac{22.4}{p}\log\left(\frac{1}{2}\right)$$
$$p\log 0.10=22.4\log\left(\frac{1}{2}\right)$$
$$p=\frac{22.4\log\left(\frac{1}{2}\right)}{\log 0.10}$$
$$p=6.7hrs$$

Question 28d

$$y = 25400(0.984)^t$$

$$20000 = 25400(0.984)^t$$

$$\frac{100}{127} = 0.984^t$$

$$\log\frac{100}{127} = \log 0.984^t$$

$$\log\frac{100}{127} = t\log 0.984$$

$$t = \frac{\log\frac{100}{127}}{\log 0.984}$$

$$t = 14.8 years$$

Question 29a

$$\log_3\left(\frac{x}{2}\right) = \log_3 11$$

$$\frac{x}{2} = 11$$

$$x = 22$$

Question 29b

$$\log_2\left(\frac{1}{3}x\right) = 4$$

$$2^4 = \frac{1}{3}x$$

$$16 = \frac{1}{3}x$$

$$48 = x$$

Question 30

See other sol'n sheet

Question 31

$$\frac{10^{5.3}}{10^x} = 125$$

$$10^{5.3-x} = 125$$

$$\log 10^{(5.3-x)} = \log 125$$

$$(5.3-x)(1) = \log 125$$

$$5.3 - \log 125 = x$$

$$x = 3.2$$